

Directive: Legibly complete these exercises; turn in problems marked “TI” for *possible* grading.

1. Create a truth table for each of the following statements, and then compute its disjunctive normal form. Which are tautological? Contradictory? Conditionally true?

(a) $(P \rightarrow Q) \rightarrow Q$

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(b) $(P \rightarrow Q) \rightarrow (Q \rightarrow P)$

(c) $(Q \leftrightarrow (\neg P)) \vee P$

TI

(d) $((P \rightarrow Q) \wedge P) \leftrightarrow (\neg Q)$

(e) $(P \vee Q) \vee (\neg Q)$

(f) $(P \wedge (\neg Q)) \vee (R \rightarrow Q)$

2. Verify each claimed logical equivalence...

i. with a truth table, and

ii. with the algebra of statements.

(a) $(P \rightarrow Q) \wedge (P \rightarrow R) \equiv P \rightarrow (Q \wedge R)$

TI

(b) $(P \rightarrow R) \wedge (Q \rightarrow R) \equiv (P \vee Q) \rightarrow R$

3. Is $p \rightarrow (q \rightarrow r)$ logically equivalent to $(p \rightarrow q) \rightarrow r$? Give a complete justification.

4. Use natural deduction to show each of the following argument forms is valid.

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(a) $X \rightarrow ((\neg Y) \rightarrow (\neg Z)) , \neg Y , Y \vee X \quad \therefore \neg Z$

(b) $X \rightarrow (X \rightarrow (\neg Y)) , Y \vee (\neg A) , X , W \rightarrow A \quad \therefore \neg W$

(c) $X \rightarrow Y , ((\neg X) \vee W) \rightarrow A , (\neg Y) \wedge Z \quad \therefore A \vee B$

TI

(d) $(X \rightarrow Y) \wedge A , (Y \rightarrow Z) \wedge B , (X \rightarrow Z) \rightarrow ((X \rightarrow Y) \rightarrow W) \quad \therefore W$

(e) $(X \wedge Y) \vee (Z \wedge W) , (X \wedge Y) \rightarrow A , (\neg A) \wedge B , Z \rightarrow (C \wedge D) \quad \therefore C$

(f) $\neg(X \wedge Y) , X , Y \vee Z \quad \therefore Z$

(g) $X \wedge (Y \vee Z) , \neg(X \wedge Y) , (Z \wedge X) \rightarrow W \quad \therefore W$

TI

(h) $X , (Y \rightarrow X) \rightarrow Z \quad \therefore X \wedge Z$

(i) $X \rightarrow Z , Y \rightarrow Z \quad \therefore (X \vee Y) \rightarrow Z$

(j) $X \rightarrow (Y \vee Z) , \neg Y \quad \therefore X \rightarrow Z$

(k) $\neg(X \vee (\neg X)) \quad \therefore Y$

5. Is the argument below valid? Give a complete (rigorous!) justification for your answer.

1. If the dog is bad or does not take a walk, then the dog cannot have a treat.

2. The dog is not sad.

3. If the dog is happy, then the dog can have a treat.

4. Thus, the dog is good.